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# Mathematical Modelling of a Fuzzy logic based Car parking system

[1](**Udo E. Umoren,** Department of Electrical and Electronic Engineering, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria) thought.umoren@gmail.com

[2](**George O. Okoronkwo,** Department of Chemical Engineering, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria) okoronkwo.george@mouau.edu.ng

[3](***Agwu E. Ogbonnaya*****,** Department of Electrical and Electronic Engineering, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria)

[4](**Chiagunye T. Theodore,** Department of Computer Engineering, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria)

**Corresponding author's email: ogeorgeonyeka@gmail.com**

***ABSTRAC :*** *This paper presents the mathematical modelling of a fuzzy logic based car parking system. Many people are facing problems on parking vehicles in parking lot in most urban cities. The technology of fuzzy based system is necessary for all vehicle users to acquire parking slot in cities. Smart parking system helps in obtaining information about available parking space, organize, process it and then place the car at certain position. The method adopted make use of a fuzzy based smart car parking system to develop the usage of small parking spaces for parking lots to allow the parking of vehicles based on their weight. Two sets of parking decisions, namely vehicle entry decision and in-park decision were used to control the weight of vehicles and the availability of parking spaces in the parking mall. The results obtained shows that the smart car parking system keeps the availability of parking lots in equilibrium depending on the entry and exit of vehicles. In conclusion, it was discovered that the total number of vehicles granted for entry at each time interval depends on the availability of parking lots at each designated floor.*

***KEYWORDS:*** *Mathematical modelling, Car parking system, Parking lot, Weight of vehicle, Fuzzy logic*

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## I. INTRODUCTION

There are large numbers of people or organization searching for parking spaces thereby contributing to traffic congestion in large cities. Again, a lot of vehicles searching for parking spaces contribute about 30% of traffic congestion experienced in large city areas (Sakthivel et al., 2020). On a daily basis, each vehicle on the road wastes 10 - 20 minutes cruising for a vacant parking spot. This causes not only a loss of time, money and effort for the drivers looking for parking space (Bhonge and Patil, 2013).

Presently, transportation fare has increasingly become an important economic, environmental and political issue. As the urban population increases, there is an increase in the density of urban mobility and this has brought about several transportation problems (Sathya and Kumaresan, 2017).

The majority growing proportion of the world's population lives in cities, with the exponential increase in car population and increase in urbanization, issues of obtaining parking places for all vehicles, efficient management of available parking lot and ensuring the security of the parking spaces have become increasingly difficult (Geng and Cassandras, 2013).

To tackle environmental challenges caused by traffic congestions as a result of searching for parking spaces in order to improve economic opportunities, numerous countries seek to improve and manage their existing transportation systems and road infrastructure to enhance traffic flow, mobility and safety. One instance of such a response is the building and deployment of guidance-based systems, such as Parking guidance and information (PGI) systems, for a better parking management (Patil and Sakore, 2014).

With the advent of internet of things (IoT) and sensing technologies, smart parking has the potential to address these challenges. Internet of Things has gone a long way to utterly altered habitual human behavior by providing them with numerous facilities and comfort options to have ease in everyday life. Equipped with an Internet connection and sensor networks, electronic devices in the digital world are connected through IoT technology (Lee, 2019).

A smart parking system (SPS) is a system that employs IoT devices or related and sensors to acquire real-time data regarding park availability so as to reduce the time required in searching for parings slots (Reve and Choudhri, 2012).

An automated parking can be designed to provide more parking spaces for the same piece of land and driver drives up to a bay at the entrance of the parking lot and leaves the car and then move the vehicle automatically to its allocated spot (Canli and Toklu, 2021).

Fuzzy rule is a form of algorithm that gives a qualitative relationship between two or more variables. This implies that it joints the input variable and its variables influence with the output variable. This rule through the relationship is as (if-then) form with vague information and a major mathematical principle that these rules are dependent on and this gives the basics of the extension principle which operate on fuzzy sets from inputs to outputs and so the work of these rules is mainly through this principle (Hikita et al., 2020).

The parking information system set the foundation for the intelligent parking service and it can provide solution to all traffic related problems. Human errors are the main source of traffic accidents, hence it is necessary to build car technologies that can check parking lot issues thereby reducing accidents and provide guidance to the parking facility.

The statement of the problem is that large numbers of people are facing problems of parking vehicles in parking lot in most urban cities. The objective is to employ a mathematical modelling of a fuzzy logic to control the weight of the vehicles and the availability of parking spaces in parking lot.

## II. MATERIALS AND METHODS

The materials used in this paper consists of the hardware and software materials to reduce time spent waiting for a free parking slot in malls and traffic congestion such as the weight in motion sensor, Arduino-UNO Controller, Arduino integrated development environment (IDE) software and MATLAB/Simulink software.
The smart car parking system utilizesthe principle of mathematical modeling through the Mamdani fuzzy inference system (MFIS).In this type of parking system, decisions must be taken by any smart car parking system from the user identifies to a parking lot on the cloud database using the internet of things. Smart car parking systems are designed to save stress and time in determining a safe parking lot for parking system.

The weight transducers are devices used to sense the weight of vehicles and convert it to an electronic form by Arduino. Again, Arduino sends this signal to the MATLAB/Simulink smart car parking system based on the fuzzy rules andthe fuzzy logic based smart car parking system served as a rule decision to the hardware component.

Again, the weight transducer detects when a vehicle enters the park and when a vehicle leaves the park. Also, the transducer collects and send record for the availability of a park lot for a light, medium or heavy weighted vehicle once a car park lot is free. The block diagram of the smart car parking system is shown in Figure 1 and Table 1 shows the weight ranges for light, medium and heavy weighted vehicles for each user.

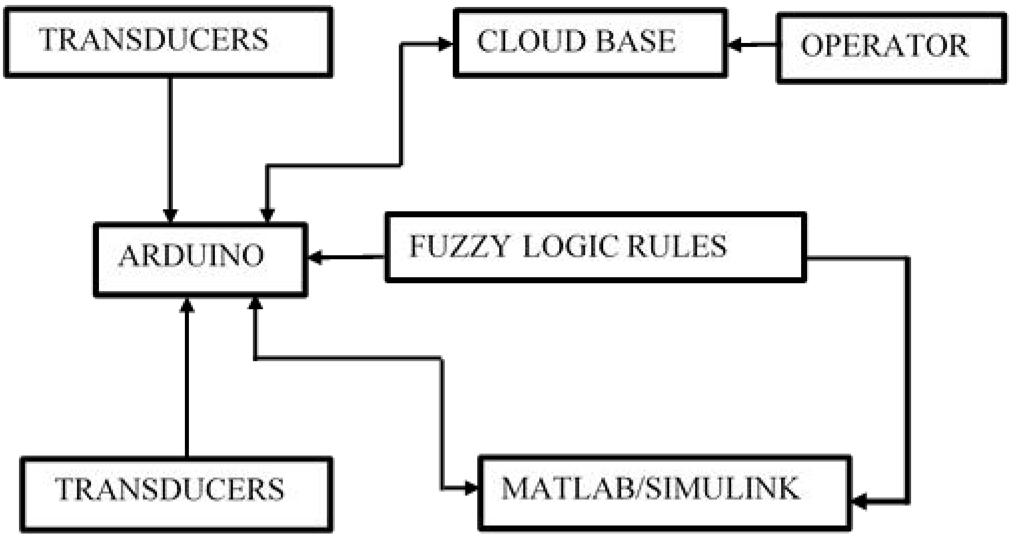


**Fig.1: Block diagram of a smart car parking system**

Table 1. Weight Ranges for Light, Medium and Heavy Weighted Vehicles for Each User

| Vehicle Type | Average weight of vehicles |
|---|---|
| Light weighted vehicles | 2000 to 4000 pounds |
| Medium weighted vehicles | 3500 to 6000 pounds |
| Heavy weighted vehicles | 5000 to 8000 pounds |

## A. MATHEMATICAL DESCRIPTION OF THE FUZZY LOGIC SYSTEM OF THE SMART SYSTEM

Let the behavior of the smart parking system during vehicle entry for n vehicles be denoted by $P_s$= i=1, 2, 3…n, then let $q$ and t indicate the availability of the presence of a free slot and the availability of none, while c, d, j indicate the presence of a light, medium and heavy weighted vehicle and $c_1$, $d_1$, $j_1$ indicate the presence of a light, medium and heavy parking lot in the garage in each of the $P_{si}$ set. Also, let $V_1$, $V_2$ and $V_3$ be:

$$V_1 = \{q, t\} \quad (1)$$

$$V_2 = \{c, d, j\} \quad (2)$$

$$V_3 = \{c_1, d_1, j_1\} \quad (3)$$

This can be attach to each smart system $P_{si}$ of the fuzzy subset, $B_i$ of $V_1$, $V_2$, $V_3$ if;

$$\{n_{ig}, n_{it}\}, \{n_{ic}, n_{id}, n_{ij}\} \ and \ \{n_{ic1}, n_{id1}, n_{ij1}\} \quad (4)$$

This equation (4) represent number of entities showing the success at each of the smart system $P_{si}$. The membership function $m_{fi}$ for each $x$ in the $V_i$ is given as:

$$m_{fV1} = \begin{cases} 1 & if \ \frac{n}{2} < n_{ix} \leq n \\ 0 & if \ 0 \leq n_{ix} \leq \frac{n}{2} \end{cases} \quad (5)$$

Also,

$$m_{fV2} = \begin{cases} 1 \quad if \ \frac{2n}{3} < n_{ix} \leq n \\ 0.5 \quad if \ \frac{n}{3} < n_{ix} \leq \frac{2n}{3} \\ 0 \quad if \ 0 \leq n_{ix} \leq \frac{n}{3} \end{cases} \quad (6)$$

Again,

$$m_{fV3} = \begin{cases} 1 \quad if \ \frac{2n}{3} < n_{ix} \leq n \\ 0.5 \quad if \ \frac{n}{3} < n_{ix} \leq \frac{2n}{3} \\ 0 \quad if \ 0 \leq n_{ix} \leq \frac{n}{3} \end{cases} \quad (7)$$

Then, the fuzzy subset $A_i$ of $V_i$ corresponding to the $P_{si}$ has the following form:

$$f_{Vi} = \{(x, m_{fV_i}(x)): x \in V\}, i = 1, 2, 3 \ldots.. n \quad (8)$$

It is necessary to represent all possible overall state of the smart system entities during the corresponding process by considering the fuzzy relation W, in $V^3$ of the form:

$$W = \{(P_s, m_V(P_s)): S = (x, y, z) \in V^3\} \quad (9)$$

## B. FUZZY INFERENCE SYSTEM FOR THE VEHICLE ENTRY DECISION

The weight of the vehicle (WOV) and availability of parking space (AOP) are the variable considered for the vehicle entry decision. The weight of the vehicle is measured by the weight transducer and the weight transducer carries out motion weight of all vehicles approaching the parking mall and sends this data to the database for availability of parking lot. The availability of parking space for the light, medium and heavy weighted vehicles was checked by the radio frequency identification (RFID) sensor that report back to the database cloud.

The membership function of weight of vehicle (WOV) is arranged in line with the triangle functions and the vehicle weight has 3 membership functions as shown in Fig.2.

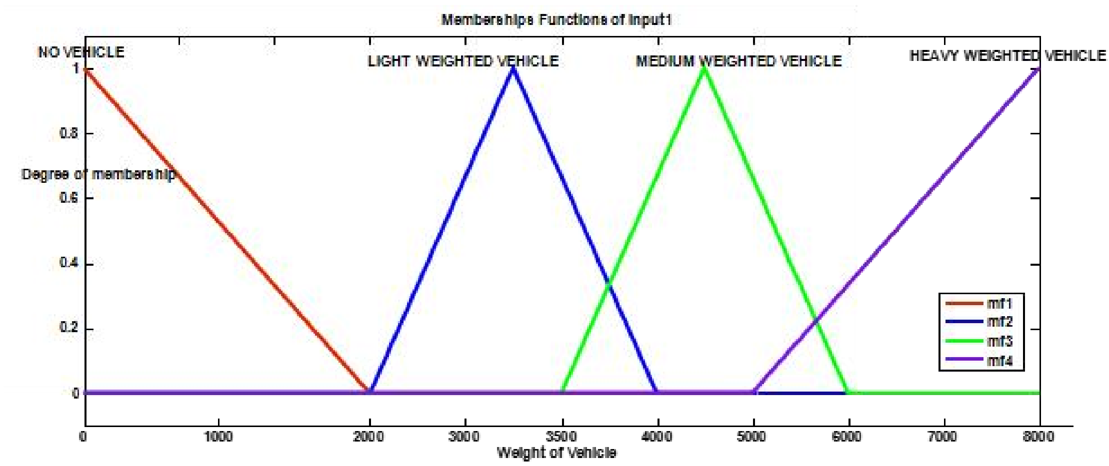


**Fig.2: Input variable 1 for weight of vehicle with membership functions**

The availability of parking space variable is composed of only two memberships as shown in Figure 3 and the fuzzy criteria for availability of parking spaces are shown in Equation (14) to (15).

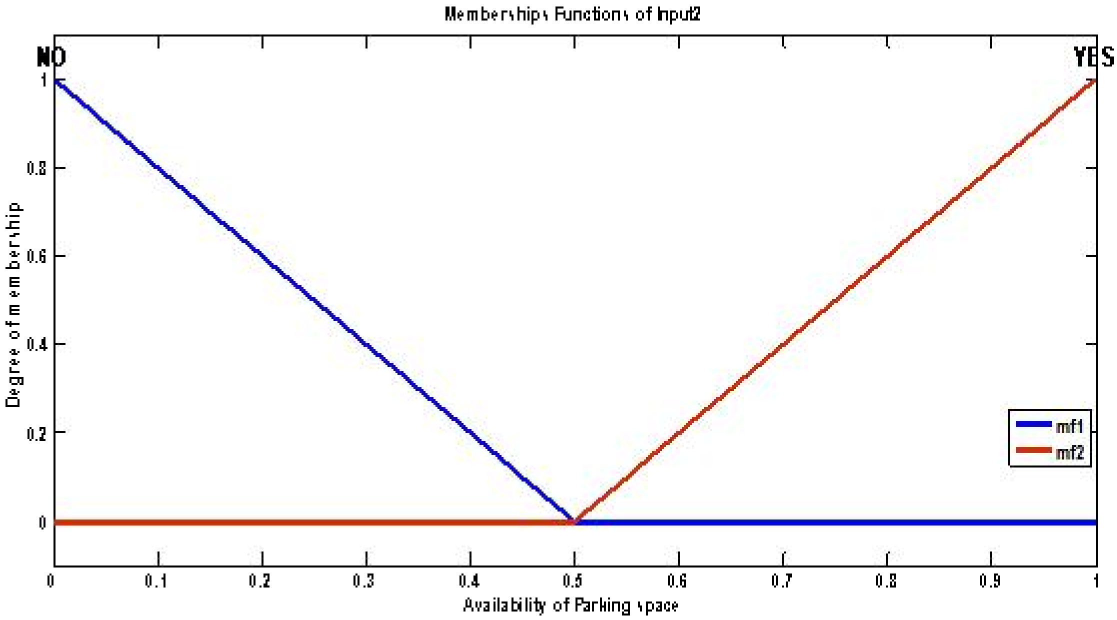


**Fig.3: Input variable 2 for availability of parking space with membership function**

The output variable is the result of the decision around the inputs, its membership functions which includes NO_ENTRY and YES_ENTRY with range of 0 to 1.

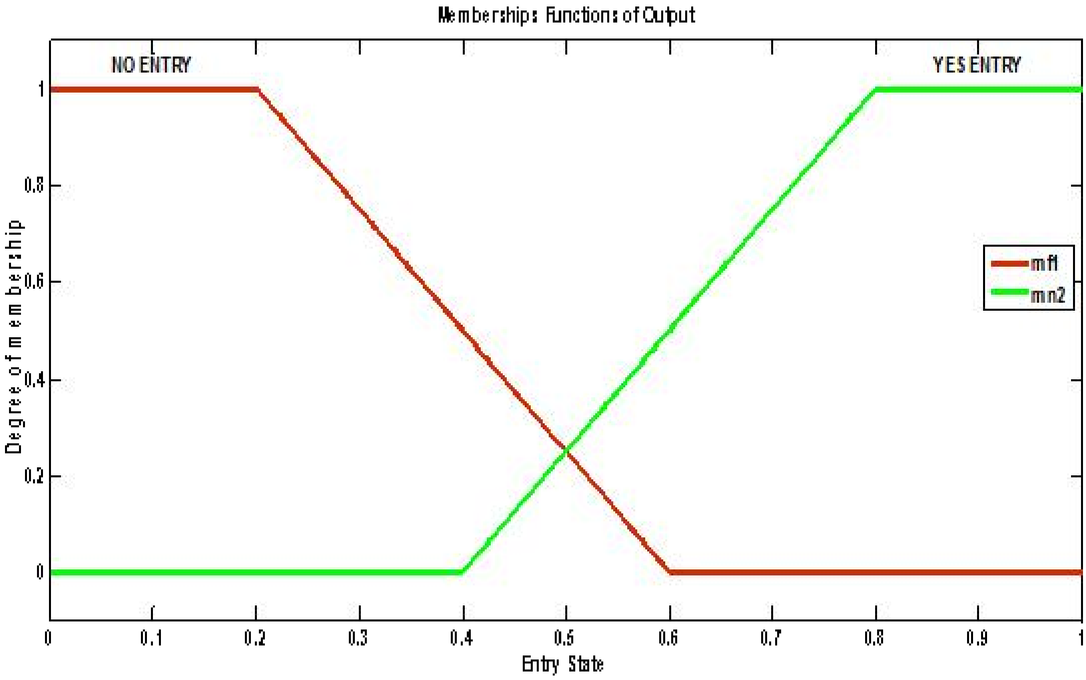


**Fig.4: Output variable for entry decision with membership functions**

$$m_{fno\_vehicle} = \begin{cases} \frac{2000 - n}{2000 - 0}; if\ 0 \leq n \leq 2000 \\ 0\ ; \quad if\ n \geq 2000 \end{cases} \qquad (10)$$

$$m_{f_{light_{weighted\_vehicle}}} = \begin{cases} 0; \quad if\ n \leq 2000\ and\ y \geq 4000 \\ \frac{n-2000}{3250-2000}; \quad if\ 2000 \leq n \leq 3250 \\ \frac{4000-n}{4000-3250}; \quad if\ 3250 \leq n \leq 4000 \end{cases} \quad (11)$$

$$m_{f_{medium\_weighted\_vehicle}} = \begin{cases} 0; \quad if\ n \leq 3500\ and\ n \geq 6000 \\ \frac{n-3500}{4250-3500}; \quad if\ 3250 \leq n \leq 4250 \\ \frac{6000-n}{6000-4250}; \quad if\ 4250 \leq n \leq 6000 \end{cases} \quad (12)$$

$$m_{f_{heavy\_weighted\_vehicle}} = \begin{cases} \frac{n-8000}{8000-5000}; if\ 0 \leq n \leq 8000 \\ 0;\ if\ n \leq 5000\ and\ n \geq 8000 \end{cases} \quad (13)$$

$$m_{fNO} = \begin{cases} \frac{0.5-n}{0.5-0}; if\ 0 \leq n \leq 0.5 \\ 0; if\ n \leq 0\ and\ n \geq 0.5 \end{cases} \quad (14)$$

$$m_{fYES} = \begin{cases} \frac{n-1}{1-0}; if\ 0 \leq n \leq 1 \\ 0; if\ n \leq 0\ and\ n \geq 1 \end{cases} \quad (15)$$

### C. FUZZY INFERENCE SYSTEM FOR THE IN-PARK VEHICLE DECISION

The in-park vehicle decision considers two input variables which are the tag number correspondence to the park slot number and the floor weight correspondence to the vehicles weight type.

The tag number correspondence for input 1 is shown in Fig. 5 and contains two membership functions and have the same fuzzy criteria as the second input variable as shown in Fig. 6.

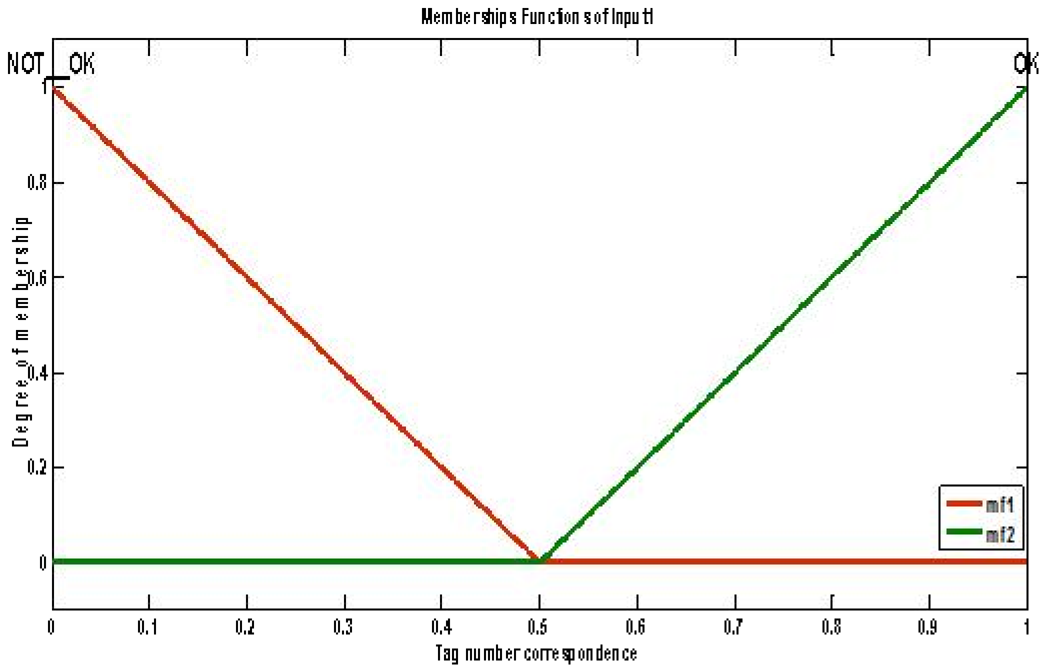


**Fig. 5: Input variable 1 for tag number correspondence with membership functions**

The floor weight correspondence for input 2 contains two membership functions as shown in Fig.6 and the fuzzy criteria are shown in equations 16 and 17.

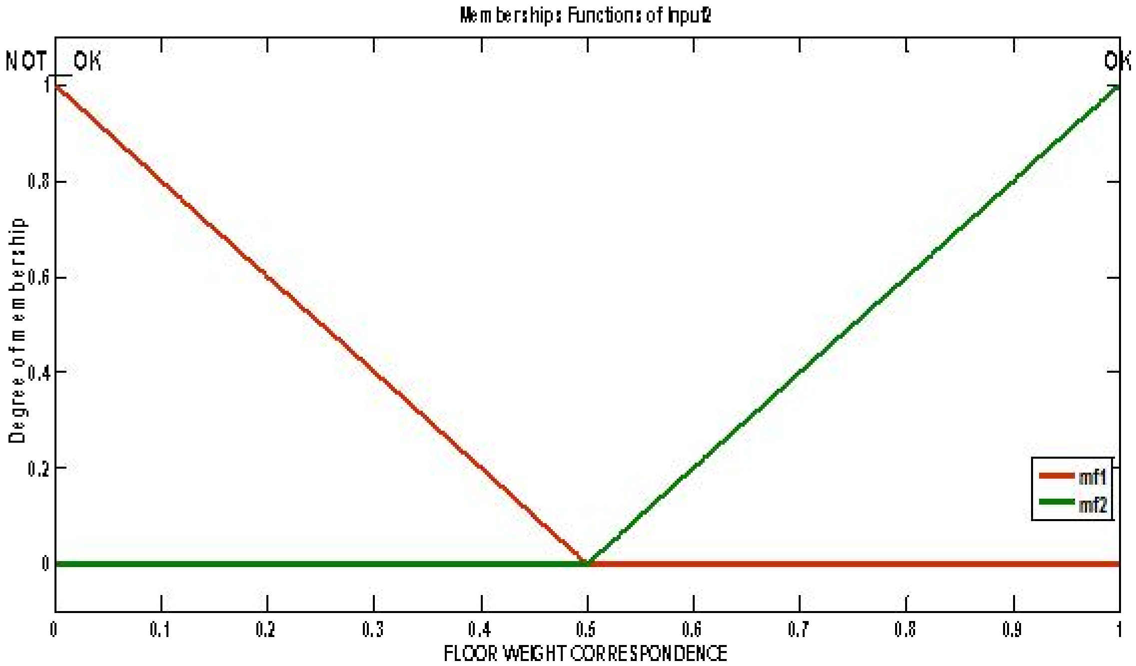


**Fig.6: Input variable 2 for floor weight correspondence with membership functions**

The output variable for entry state is the result of the decision for the inputs and its membership functions includes ENTRANCE_REMAINS_CLOSED and OPEN_ENTRANCE with range of 0 to 1 as described in Fig. 7.

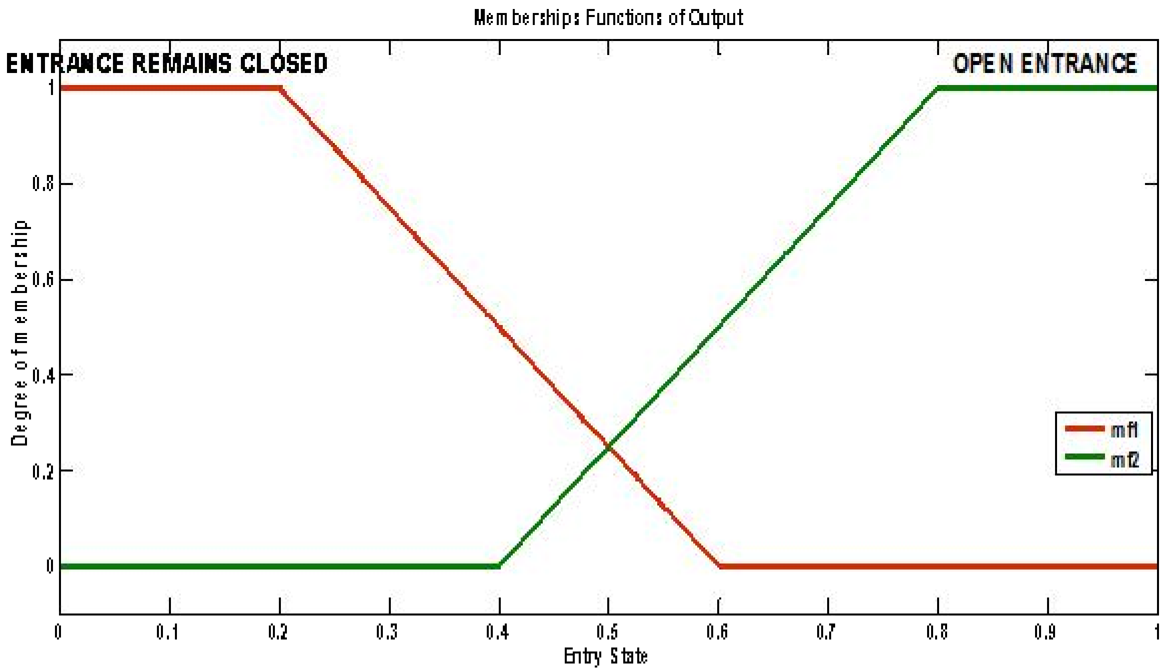


**Fig.7: Output variable of in-park decision with membership functions**

$$m_{fNOT\_OK} = \begin{cases} \frac{1-n}{1-0} ; if\ 0 \leq n \leq 1 \\ 0 ; if\ n \geq 1 \end{cases} \quad (16)$$

$$m_{fOK} = \begin{cases} \frac{n-1}{1-0} ; if\ 0 \leq n \leq 1 \\ 0 ; if\ n \leq 0\ and\ n \geq 1 \end{cases} \quad (17)$$

## III. RESULTS AND DISCUSSION

The results realized from the vehicle entry decision and in-park entry decisions are shown in Figs. 8, 9 and 10. The results obtained provide information about the vehicle entry and exit based on the vehicle weights and the availability of parking lots for each vehicle.

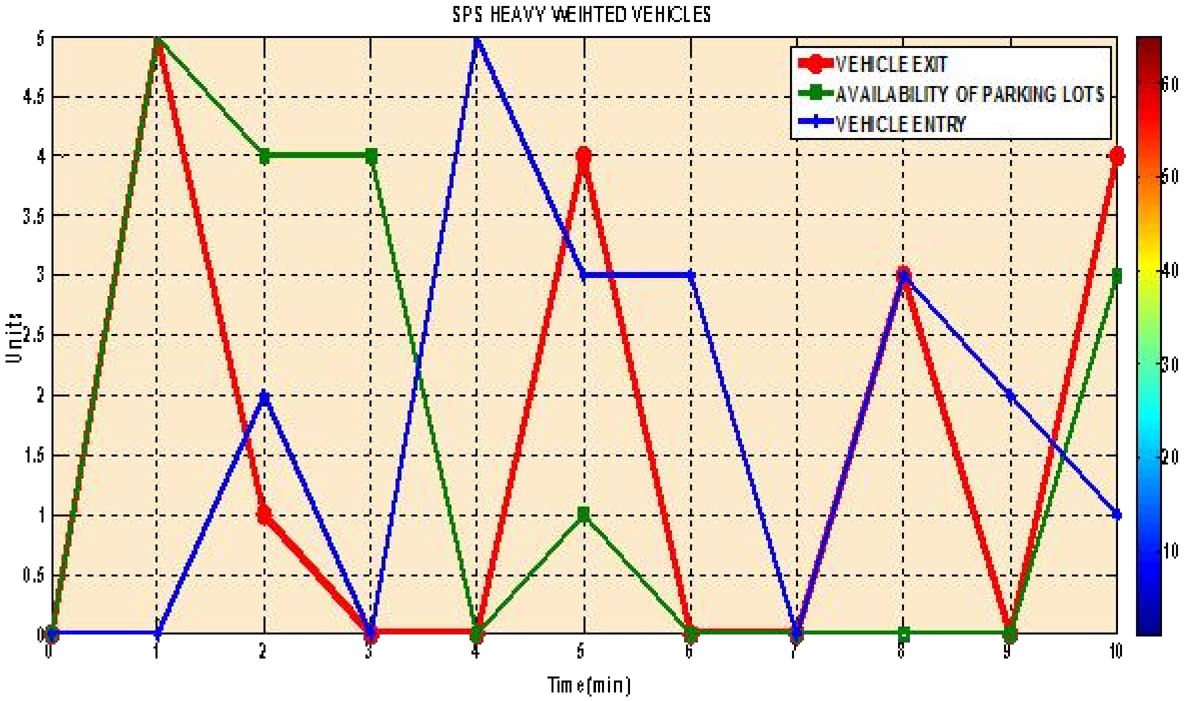


**Fig.8: Plots showing vehicle entry, exit and availability of parking lots for heavy weighted vehicles**

In Fig. 8, when the availability of parking lots is in equilibrium, it is observed that there is a slight difference between the vehicle entry and vehicle exit each time a vehicle enters and leaves the smart system. For instance, the plot shows that when the heavy weighted vehicle was at four minutes, the vehicle exits and availability of parking lots was very low due to large entry of vehicles and as a result the smart system denies access to other vehicles to keep the system in equilibrium. Also, when the vehicle was at six minutes, vehicle exit took place at zero unit, vehicle entry at three units and availability of parking lots at zero unit.

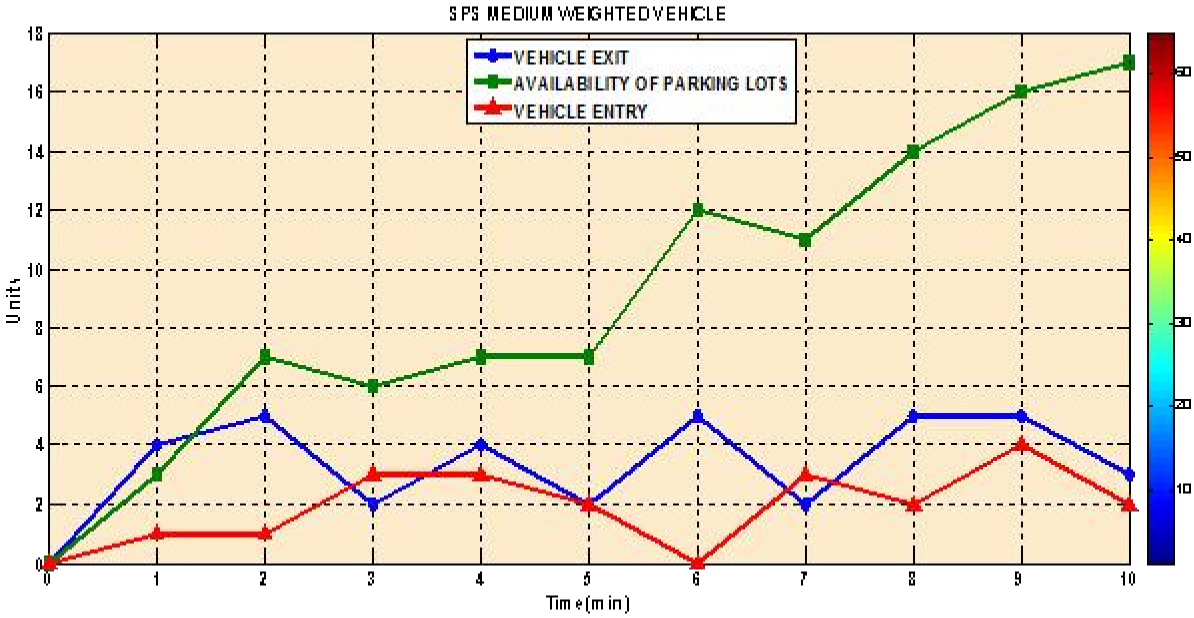


**Fig. 9: Plots showing vehicle entry, exit and availability of parking lots for medium weighted vehicles**

In Fig. 9, it is observed that the smart parking system ensures that the vehicle entry does not exceed the availability of parking lots because the vehicle exit is more than the vehicle entry. For instance, when the vehicle was at nine minutes, vehicle exit occurs at five units, vehicle entry at four units and the availability of parking lots at sixteen units.

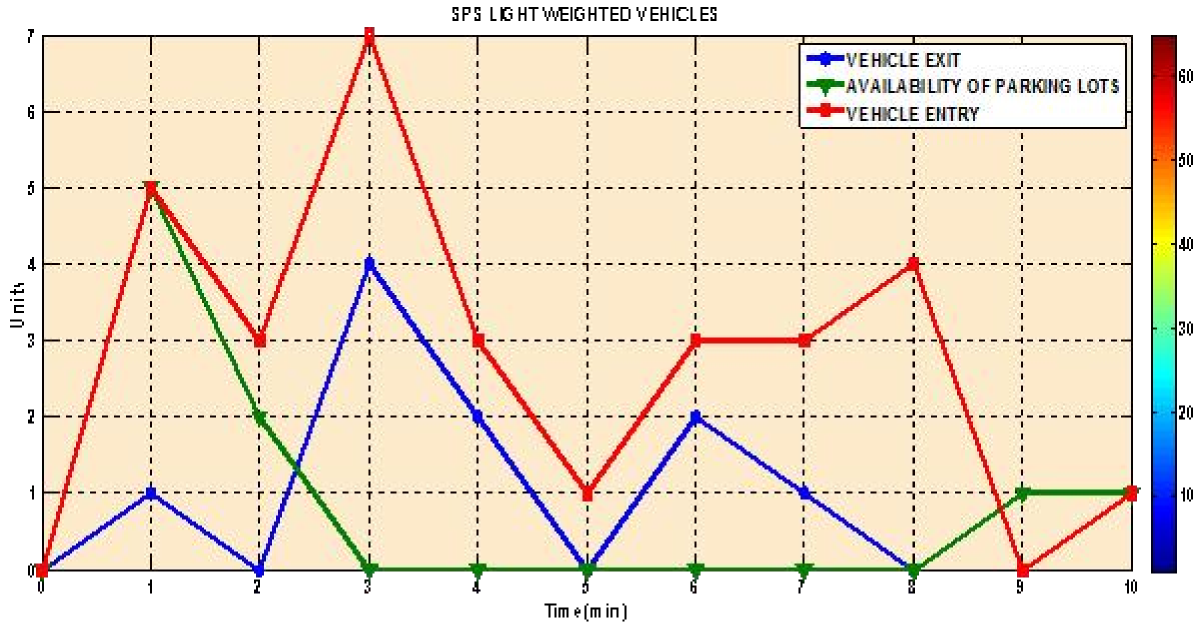


**Fig. 10: Plots showing vehicle entry, exit and availability of parking lots for light weighted vehicles.**

In Fig. 10, it is observed that when the vehicle entry exceeds the vehicle exit and as a result the availability of parking lots remains less until the vehicle exit exceeds the vehicle entry for light weighted vehicle.

For instance, when the vehicle was at four minutes, vehicle exit occurred at two units, vehicle entry at three units and the availability of parking lots remain at zero unit, Also, when the vehicle was at eight minutes, vehicle exit take place at zero unit, vehicle entry occurs at four units and the availability of parking lots remain at zero unit and this indicates that there was no space for parking lots.

## IV. CONCLUSION

To overcome the problems of parking vehicles on parking lot in some cities, the automatic vehicle smart parking system is the main idea in the traffic and this system can be controlled without human interference. The electronic data keeps information of the available number of parking lot using the transducer and the transducer collects and post records of the available parking lot for the light, medium and heavy weighted vehicles once a park lot is free. The vehicle detection at the entry can be achieved when using the weight transducer to detects when a vehicle tries to enter the park and when a vehicle leaves the park. The availability of the parking lot for a sensed weighted vehicle and the availability of the parking lot depend on the presence of the RFID reader of different categories of vehicles.

**Conflict of interest**

The authors declare that there is no conflicting in the publication of this paper

**Recommendations**

It is recommended that MATLab toolbox other than fuzzy logic should be used to design and model car parking system in order to compare the results obtained from it with that of the fuzzy logic.

**Acknowledgement**

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