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Formulation of Mathematical Model for Maximum Shear Strain (Distortion) Energy Theory of Yield for Plane Continuum

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ABSTRACT : *The failure of engineering structures can have catastrophic consequences, leading to significant economic losses—both material and financial—and, in some cases, loss of human lives. Such failures often occur due to inadequate determination of structural loads or when materials exceed their yield strength, causing excessive deformation. This study revisits von Mises' theory and presents a newly formulated general applied stress mathematical model. Polynomial displacement shape profiles were employed to evaluate n-values for different plate types, leading to the derivation of n-value equations. Additionally, new general applied stress equations, Stress Factor (Fss), equations for shear strain energy theory for various boundary conditions, and allowable stress equations were developed. Validation of the newly formulated equations revealed that the applied stress values exceeded the yield stress of structural steel for plates with one free edge. To mitigate potential failure, a safety factor of 1.15 was introduced for the plate types considered which reduces the applied stress of 281N/mm2 to allowable stress of 244N/mm2 below the material yield stress of 250N/mm2. The results obtained align with findings from existing literature, confirming the reliability of the developed equations. As such, the new equations provide an effective means of predicting the allowable stress of plane materials based on the maximum shear strain energy theory.*

KEYWORDS *Structural Failure, Applied Stress, Shear Strain Energy Theory, Polynomial Displacement Profiles, Factor of Safety*

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I. INTRODUCTION

The study of material failure theories plays a crucial role in predicting the behaviour of engineering materials under various loading conditions. One of such theory, the Maximum Shear Strain (Distortion) Energy Theory, commonly known as the von Mises yield criterion, is employed in the evaluation of the yielding point of ductile materials. Understanding this theory is essential for ensuring the safety and reliability of engineering structures subjected to multi-axial stress conditions. The theory asserts that failure occurs when the distortion energy per unit volume reaches a critical value, a concept that is particularly important in the design and analysis of continuum structures.

Strain commonly referred to as deformation, is the ratio of change in length to the original length of a material. According to Hosford (2005), bodies undergo translations and rotations as they deform, as such strain must be defined in such a way to exclude the effects of translation and rotation. Strain that disappears instantaneously upon the release of force is termed elastic strain (Meyers M. A. and Chawla K. K., 2009).

The octahedral shear stress theory, initially proposed by Hacky and improved by Von Mises is usually called Hacky and Von Mises yield Criterion.

The challenge with octahedral shear stress theory and maximum stress theory is that, Where is shear stress at yield

But torsional test on mild steel specimens have found that

The implication here is that the octahedral shearing stress theory is not always suitable and this is because of the fact that it ignores the effects of principal stress along y-axis, and Poisson ratio,. Maximum shear strain energy theory takes into consideration all these factors and does not fail under the hydrostatic stress condition. (Ross, 1987).

The formulation of a mathematical model for maximum shear strain energy theory of yield for plane continuum is essential for advancing material strength analysis. As noted by Liu et al. (2019), optimizing stress distribution based on distortion energy principles can significantly enhance the structural integrity of materials. Furthermore, Bhat et al. (2018) emphasized the importance of this theory in evaluating multi-axial fatigue in ductile materials, particularly through its application in static state stress analysis and modified Goodman relationships. According to Onaka (2010), maximum shear strain energy theory is an appropriate measure for large deformation. Despite the extensive research and applications of failure theories, a comprehensive mathematical model specifically addressing the maximum shear strain energy in plane continuum structures remains underexplored.

B. LITERATURE REVIEW

The von Mises theory was originally proposed by Huber (1904) and later refined by von Mises (1913). Shrivastava et al. (2012) derived the von Mises equivalent strain increment for the case of large strain simple shear. According to Karmankar et al. (2017), the von Mises stress is a crucial parameter in determining the yield point of ductile materials. A strain-energy-based method has been employed for the prediction of fatigue life of a structure (Emuakpor et al., 2010). The significance of the theory is further supported by Kosaroglu and Khalikov (2009), who formulated strain energy equations that quantify material deformation under different stress states.

Recent studies have extended the application of distortion energy theory to modern materials and loading conditions. Jin et al. (2022) investigated strain rate effects on rebar and concrete materials, highlighting the growing importance of strain considerations in structural engineering. Additionally, Pardis et al. (2017) introduced a novel definition of "true shear strain," providing a refined approach for evaluating effective strain in shear deformation. Okajima et al. (2001) established maximum shear strain and earth pressure distribution in an attempt to clarify the rotating failure of a retaining structure with excavation by finite element analysis with the implicit dynamic relaxation method. Zhang et al. (2011) investigated the effect of strain reversal on hardening due to high pressure torsion (HPT) using commercially pure aluminium.

Maximum shear strain energy theory as presented by Sutar (2025), "the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e, distortion energy at yield point) per unit volume as determined from a simple tension test". Furthermore, experimental validation through equivalent stress methods and Goodman's criterion, as outlined by Bhat et al. (2018), reinforces the reliability of this theory in practical engineering applications.

Despite the advancements in failure theory research, there remains a gap in the formulation of a mathematical model specifically addressing maximum shear strain energy in plane continuum structures. This study aims to bridge that gap by developing a comprehensive mathematical model for the maximum shear strain energy theory of yield, thereby contributing to safer and more efficient structural designs.

II.. MATHEMATICAL FORMULATION

This theory states that elastic failure takes place when the shear strain energy per unit volume, at a point, equals to the shear strain energy per unit volume in a specimen of the material, in the simple uniaxial test.

According to Ross (1987), the shear strain energy (SSE)/vol. = Total strain energy/vol -Hydrostatic strain energy/vol.

$$\mathrm{SSE} = \mathrm{U_{T}} - \mathrm{U_H} \qquad (3)$$

Recall

Total strain, $U_T = \frac{1}{2E}[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\upsilon(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)] * \text{volume}$ (4)

Hydrostatic strain energy,

$U_H = \frac{1}{2E}(3\sigma_H^2 - 2\upsilon * 3\sigma_H^2) \text{ x volume}$ (5)

$U_H = \frac{\sigma_H^2}{2E}(3 - 6\upsilon) * \text{volume}$ (6)

Where hydrostatic stress,

$\sigma_H = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$ (7)

Substituting equation (7) into equation (6) yields

$$U_H = \frac{(\sigma_x + \sigma_y + \sigma_z)^2}{2E * 3^2}(3 - 6\upsilon) * \text{volume} \quad (8)$$

$$U_H = \frac{3(1 - 2\upsilon)}{2E * 3^2}(\sigma_x + \sigma_y + \sigma_z)^2 * \text{volume} \quad (9a)$$

$$U_H = \frac{(1 - 2\upsilon)}{6E}(\sigma_x + \sigma_y + \sigma_z)^2 * \text{volume} \quad (9b)$$

Substituting equations (9b) and (4) into equation (3) yields

$$SSE/vol. = \frac{1}{2E}[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\upsilon(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)] - [\frac{(1 - 2\upsilon)}{6E}(\sigma_x + \sigma_y + \sigma_z)^2] \quad (10)$$

$$SSE/vol. = \frac{1}{2E}[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\upsilon\sigma_x\sigma_y - 2\upsilon\sigma_x\sigma_z - 2\upsilon\sigma_y\sigma_z] + \frac{(2\upsilon - 1)}{6E}(\sigma_x + \sigma_y + \sigma_z)^2$$

Simplifying further yields

$SSE/vol. = \frac{2(1+\upsilon)}{6E}\frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2]$ (11)

But $G = \frac{E}{2(1+\upsilon)}$ (12)

$SSE/vol. = \frac{1}{12G}[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2]$ (13)

For uniaxial tensile test

$\sigma_x = f_y, \sigma_2 = \sigma_3 = 0$ (14)

Therefore, equation (13) becomes

$$SSE/vol. = \frac{1}{12G}[f_y^2 + f_y^2]$$

$$SSE/vol. = \frac{f_y^2}{6G} \quad (15)$$

Comparing Equations (13) and (15) yield

$\frac{1}{12G}[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2] = \frac{f_y^2}{6G}$

$[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2] = 2f_y^2$ (16)

For a plane stress case

$\sigma_z = 0$

$(\sigma_x - \sigma_y)^2 + \sigma_x^2 + \sigma_y^2 = 2f_y^2$ (17)

$\sigma_x^2 - 2\sigma_x\sigma_y + \sigma_y^2 + \sigma_x^2 + \sigma_y^2 = 2f_y^2$

$$2\sigma_x^2 + 2\sigma_y^2 - 2\sigma_x\sigma_y = 2{f_y}^2$$

$$\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 = {f_y}^2 \quad (18)$$

$$\sigma_x^2\left(1 - \frac{\sigma_y}{\sigma_x} + \frac{\sigma_y^2}{\sigma_x^2}\right) = {f_y}^2 \quad (19)$$

$$\sigma_x^2(1 - m_1 + m_1^2) = {f_y}^2 \quad (20)$$

Where $m_1 = \frac{\sigma_y}{\sigma_x}$ (21)

Another interpretation of Equation (17) is that elastic failure occurs in a structure when the octahedral shear stress at a point in it reaches the octahedral shear stress at yield, in a specimen made from the same material as the structure, when the specimen undergoes the simple uniaxial test.

Recall from Adah et al. (2023)

$$\sigma_x = -\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{\beta^2}\frac{\partial^2 h}{\partial Q^2}\right) \quad (22a)$$

$$\sigma_y = -\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\upsilon\partial^2 h}{\partial R^2} + \frac{1}{\beta^2}\frac{\partial^2 h}{\partial Q^2}\right) \quad (22b)$$

Substitute Equation (22) into Equation (21) yield

$$m_1 = \frac{\frac{\upsilon\partial^2 h}{\partial R^2} + \frac{1}{\beta^2}\frac{\partial^2 h}{\partial Q^2}}{\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{\beta^2}\frac{\partial^2 h}{\partial Q^2}} \quad (23)$$

$$m_1 = \frac{n_2}{n_1} \quad (24)$$

Where

$$n_1 = \frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{\beta^2}\frac{\partial^2 h}{\partial Q^2} \quad (25)$$

$$n_2 = \frac{\upsilon\partial^2 h}{\partial R^2} + \frac{1}{\beta^2}\frac{\partial^2 h}{\partial Q^2} \quad (26)$$

Substitute Equation (25) and (26) into Equation (20) yields

$$\sigma_x^2\left(1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}\right) = {f_y}^2 \quad (27)$$

$$\sigma_x^2 = \frac{{f_y}^2}{[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}]} \quad (28)$$

$$\sigma_x = \frac{f_y}{\sqrt{[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}]}} \quad (29)$$

$$\sigma_x = \frac{f_y}{F_{ss}} \quad (30)$$

Where

$$F_{ss} = \sqrt{[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}]} \quad (31)$$

A. Evaluation of 'n-Values' and Formulation of 'n-Values' Equations

The various n-values (that is, n_1, n_2) for the different plate types will be evaluated using the polynomial displacement shape profiles in Table 1.

Table 1: The polynomial displacement shape profiles

Plate Type	Shape Profile, h
CSFS	$(R-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
SSFS	$(R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
SCFS	$(1.5R^2-2.5R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CCFS	$(1.5R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
SCFC	$(R^2-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CCFC	$(R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

(Ibearugulem et al. 2014)

S-Simply supported edge, C - Clamped edge, F - Free edge

Where

CSFS - a plate clamped at edge 1, simply supported at edges 2 and 4 and free at edge 3.

CCFC- a plate clamped/fixed on edges 1,2 and 4, and free on edge 3.

$R = X/a$, $0 \le R \le 1$; $Q = Y/b$, $0 \le R \le 1$

a - plate dimension (length) along X-axis, b - is plate dimension (Width) along Y-axis

The n-values for the various plate types will be evaluated as follows.

Evaluation of n-Values for CSFS Plate

From Table 1,

Evaluation of n-values

$$h = (R - 2R^3 + R^4)(2.8Q^2 - 5.2Q^3 + 3.8Q^4 - Q^5) = h_x * h_y \qquad (32)$$

$$\frac{\partial^2 h}{\partial R^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y = (-12R + 12R^2)(2.8Q^2 - 5.2Q^3 + 3.8Q^4 - Q^5) \qquad (33a)$$

$$\frac{\partial^2 h}{\partial Q^2} = h_x * \frac{\partial^2 h_y}{\partial Q^2} = (R - 2R^3 + R^4)(5.6 - 31.2Q + 45.6Q^2 - 20Q^3) \qquad (33b)$$

$$\frac{\partial^2 h}{\partial R \partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} = (1 - 6R^2 + 4R^3)(5.6Q - 15.6Q^2 + 15.2Q^3 - 5Q^4) \qquad (33c)$$

Substitute Equations (33)in Equations (25) and (26) yields

$$n_1 = \Big[(-12R + 12R^2)(2.8Q^2 - 5.2Q^3 + 3.8Q^4 - Q^5) + \frac{\upsilon}{ƹ^2}(R - 2R^3 + R^4)(5.6 - 31.2Q + 45.6Q^2 - 20Q^3)\Big] \qquad (34)$$

$$n_2 = \Big[\upsilon(-12R + 12R^2)(2.8Q^2 - 5.2Q^3 + 3.8Q^4 - Q^5) + \frac{1}{ƹ^2}(R - 2R^3 + R^4)(5.6 - 31.2Q + 45.6Q^2 - 20Q^3)\Big] \qquad (35)$$

At the point of maximum deflection, R = 0.5, Q = 1. Substitute these values of R and Q in Equation (34) to Equation (35), we have

$$n_1 = -1.2 \qquad (36)$$

$$n_2 = -1.2\upsilon \qquad (37)$$

Substituting Equation (36) and Equation (37) into Equation (31) yields the stress factor as Equation (38)

$$F_{ss} = \left[1 - \frac{(-1.2\upsilon)}{(-1.2)} + \frac{(-1.2\upsilon)^2}{(-1.2)^2}\right]^{\frac{1}{2}}.$$

$$F_{ss} = [1 - \upsilon + \upsilon^2]^{\frac{1}{2}}. \qquad (38)$$

Similarly, the rest of the five plate types contain in Table 1, were evaluated.

The stress factor Equation (38) for the CSFS plate and other plate types under consideration are presented in Table 3.

Also, using trigonometric shape profile.

From Ibearugbulem et al. (2019), the shape profile for CSFS is

$$h = \mathrm{Sin}(m\pi R)\left(\mathrm{Cos}(\frac{n\pi}{2}Q) - 1\right) = h_x * h_y \qquad (39)$$

$$\frac{\partial^2 h}{\partial R^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y = -m^2\pi^2 \mathrm{Sin}(m\pi R)\left(\mathrm{Cos}\left(\frac{n\pi}{2}Q\right) - 1\right) \quad (40a)$$

$$\frac{\partial^2 h}{\partial Q^2} = h_x * \frac{\partial^2 h_y}{\partial Q^2} = \mathrm{Sin}(m\pi R)\left(-\frac{n^2\pi^2}{4}\mathrm{Cos}\left(\frac{n\pi}{2}Q\right)\right) \quad (40b)$$

$$\frac{\partial^2 h}{\partial R \partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} = m\pi \mathrm{Cos}(m\pi R)\left(-\frac{n\pi}{2}\mathrm{Sin}\left(\frac{n\pi}{2}Q\right)\right) \quad (40c)$$

Substitute Equations (40) into Equations (25) and (26) yields

$$n_1 = \left[\left(-m^2\pi^2 \mathrm{Sin}(m\pi R)\right)\left(\mathrm{Cos}\left(\frac{n\pi}{2}Q\right) - 1\right) - \frac{\upsilon 4 n^2\pi^2}{4ƨ^2}\mathrm{Sin}(m\pi R)\mathrm{Cos}\left(\frac{n\pi}{2}Q\right)\right] \quad (41)$$

$$n_2 = \left[\upsilon\left(-m^2\pi^2 \mathrm{Sin}(m\pi R)\right)\left(\mathrm{Cos}\left(\frac{n\pi}{2}Q\right) - 1\right) - \frac{4n\pi^2}{ƨ^2}\mathrm{Sin}(m\pi R)\mathrm{Cos}\left(\frac{n\pi}{2}Q\right)\right] \quad (42)$$

At the point of maximum deflection, R = 0.5, Q = 1. Substitute these values of R and Q in Equation (41) and Equation (42), we have

$n_1 = 9.869604404$ (43)

$n_2 = 9.869604404\upsilon$ (44)

Substituting Equation (43) and Equation (44) into Equation (31) yields the stress factor as Equation (45)

$$F_{ss} = \left[1 - \frac{9.869604404\upsilon}{9.869604404} + \frac{(9.869604404\upsilon)^2}{9.869604404^2}\right]^{\frac{1}{2}}.$$

$$F_{ss} = \left[1 - \upsilon + \upsilon^2\right]^{\frac{1}{2}}. \quad (45)$$

Numerical Application

Consider a structural steel square plate with the following properties. $\upsilon = 0.3$, a =1m, f_y =250MPa.
The numerical results obtained from yield criterion equations in Table 3 are presented in Table 4.

III. RESULTS AND DISCUSSION

The new general applied stress equation obtained from the present study is presented in Table 2. The stress factor, F_{ss}, equations for shear strain energy theory for the different boundary conditions are given in Table 3. The allowable stress equation from this work is given in Table 4. This is obtained by dividing the applied stress by a factor of safety.

Table 2: General Applied Stress Equation

SN	DESCRIPTION	EQUATIONS
1	Applied Stress	$\sigma_x = \frac{f_y}{F_{ss}}$
2	Stress Factor of Safety	$F_{ss} = \sqrt{\left[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}\right]}$
3	n-values for plates	$n_1 = \left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$
		$n_2 = \left(\upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$

Table 3: Stress Factor, F_{ss}, Equations for shear strain energy theory

Plate Type	$\sigma_x = \frac{f_y}{F_{ss}}$
	$F_{ss} = \sqrt{\left[1 - \frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}\right]}$
SSFS	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
SCFS	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
CSFS	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
CCFS	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
SCFC	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$
CCFC	$[1 - \upsilon + \upsilon^2]^{\frac{1}{2}}$

Table 4: Shear Strain Allowable Stress from Failure Criterion Analysis

Plate Type	$\sigma_{allow} \leq \frac{\sigma_x}{F}$; $\varepsilon = \frac{b}{a} = 1$; $f_y = 250MPa$; $F = 1.15$		
	F_{ss}	σ_x	σ_{allow}
CSFS	0.88881944	281	244
SSFS	0.88881944	281	244
SCFS	0.88881944	281	244
CCFS	0.88881944	281	244
SCFC	0.88881944	281	244
CCFC	0.88881944	281	244

From column 3 of Table 4, the applied stress values are higher than the yield stress of structural steel for plates with one free edge. This is because the stress factor Fss for shear strain theory is less than unity. This implies that the plate with one free edge is possibly going to fail, since the applied stress is greater than the yield stress of the material. In order to avert the potential failure, the applied stress is divided by a factor of safety to reduce it below the material yield stress. For the case, the factor of safety chosen is 1.15. This is because 1.15 is within the limit of the factor of safety specified by BS8110 and BS 5950. This now gave rise to the allowable stress value which is less than the yield stress of structural steel as shown in column 4 of the Table 4. Comparing these values with earlier work by Adah et al (2025) who used octahedral shear stress theory indicates that, the results are the same even though a different approach or theory was used and the equations are different. This validates these new equations for different plate boundary conditions and implies that the new equations are adequate for predicting the allowable stress of a plane material based on maximum shear strain energy theory. It has also improved the adequacy of the von Mises yield criterion as its applies to plane continuum especially, plates. More so, Equation (38) obtained from polynomial shape function and Equation (45) obtained from trigonometric shape function gave the same equation. This further highlights the adequacy of the present approach.

IV. CONCLUSION

The maximum shear strain energy theory or von Mises theory has been revisited and looked at from the plane continuum perspective especially plates with one free edge. Based on this theory, a new general applied stress mathematical model has been formulated. Based on this new model stress factors for the different plates considered here were evaluated and found to be all less than unity which resulted in a stress greater than yield stress of structural steel signifying potential failure. To avert this, a new model was proposed by introducing a factor of safety of 1.15 which yielded an allowable stress less than the yield stress. This is then considered adequate.

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