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MATHEMATICAL MODELS FOR LIMIT STATE AMPLITUDE OF DISPLACEMENT ANALYSIS OF RECTANGULAR PLATES

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ABSTRACT: *Limit State Amplitude of Displacement is the limiting point or value of the amplitude of displacement beyond which the possibility of severe damage or failure is prominent. Understanding this level or point is key to determining the safety of plate structures. This work aims to derive the limit state expression that will help predict this limiting point or value for plate structures. The yield stress criterion equation was established. The yield stress was substituted with the stress equation in a plane using non-dimensional parameters. The displacement shape function, w , in the equation, was replaced by the product of the amplitude of displacement, A, and the shape profile, h, and the equation evaluated by making, A, the subject of the equation. The resulting equation became the general limit state amplitude of displacement expression. Twelve plate types were considered and their shape profiles were evaluated and substituted into the general limit state amplitude of displacement equation to obtain the specific limit state amplitude of displacement expression for each plate type. Numerical applications were carried out to obtain the numerical values of the coefficient of the amplitude of displacement and the limit state amplitude of displacement for each plate type. From the new limit state equations, it was observed that the limit state amplitude of displacement is directly proportional to the length of the plate and inversely proportional to the thickness of the plate. This implies that the longer the plate's span the higher the amplitude of displacement, and the higher the thickness of the plate the lesser the amplitude of displacement. This relationship observed from the new equations conforms with the actual and practical behavior of structural elements such as plates, confirming the adequacy of the new equations. More so, the approach is easy and the equation can easily predict the amplitude of displacement of plates once the length of the plate and its thickness is known with the plate type determined. The work will help analysts and designers of plates with easy data to determine the limit of safety of plate structures based on amplitude of displacement, since the higher the amplitude of displacement, the higher the chances of failure or severity of damage. Hence, the conclusion is that these new equations are adequate for thin rectangular plate analysis for the amplitude of displacement and that the approach is simpler, understandable, and reproducible.*

KEYWORDS: *Amplitude, Displacement, Limit state, Shape profiles, Rectangular plates*

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I. INTRODUCTION

The limit state is the state beyond which a structure loses its functionality. It is the ultimate point of a structural capacity beyond which it is assumed to have failed. Amplitude describes the severity of the disturbance associated with a wave. It is the maximum displacement from equilibrium (www.brightstorm.com>vibrations and waves>a…). Every structure vibrates or oscillates when loaded. The oscillation limit is called the amplitude of displacement of that structure. The higher the amplitude, the more severe the wave is, and the more the disturbance is. Also, the amplitude of a wave is related to the amount of energy it carries, such that, the higher amplitude wave carries a bigger amount of energy and vice versa (https//dosits.org>science>sound> characterize-sounds). For the safety of structures, this energy must be checked. This means the amplitude must be checked in order not to exceed its safe or allowable limit. This is very important because structures (plates inclusive) are generally designed for both ultimate and serviceability limit states as specified in codes such as British Standards, Euro codes, etc. And the key element of the serviceability limit state is deflection which is a function of the amplitude. The amplitude of displacement 'A' is the ratio of the displacement 'w' to the shape profile, h (Ibearugbulem et al. 2014; Ogbuahamba et al, 2015; Onwuka et al. 2016, Onodagu, 2018).

For a line continuum such as a beam, column, etc, the shape profile h, is one-dimensional. But for a plane continuum such as plates, the shape profile, h, is two-dimensional (Ibearugbulem, 2012). The focus of this work is on thin rectangular plates. The amplitude of displacement can be small or large based on the magnitude of displacement. It is called small amplitude when the displacement is within the elastic range but when the displacement is within the inelastic range it is then large amplitude. For a thin plate, the amplitude becomes large when the displacement is greater than or equal to the plate thickness. Large amplitude results in the stretching of the middle fiber and the middle fiber strains are no longer small and negligible (Chajes, 1974; Iyengar, 1998; Ventsel and Krauthermer, 2001). The large amplitude of displacement leads to resonance. Resonance is large amplitude vibration caused by a small periodic stimulus having the same or nearly the same, period as the system's or element's natural vibration (www.sciencedirect.com>topics>engineering).

The amplitude of displacement varies from one structural boundary condition to the other. The nature of the boundary condition affects the magnitude of the amplitude of displacement. Various Scholars have expressed the shape profile in terms of trigonometric, polynomial, or hyperbolic functions (Levy, 1942; Timoshenko, 1959; Ugural, 1999; Ventsel and Krauthermer, 2001; Chakarverty, 2001; Szilard, 2004, Ibearugbulem et al. 2014). The trigonometric and hyperbolic functions are very difficult to evaluate. Hence, Ibearugbulem (2012) in his PhD thesis formulated the deflected shape profile using polynomial series, which gave rise to the polynomial shape profiles. These polynomial shape profiles are easy to evaluate and will be used in this work. For small deflection of plates, the amplitude of displacement has been expressed by Adah et al. (2016) for aspect ratio

$Ƨ = b/a$ as

$$A = u_s \frac{qa^4}{D} \tag{1}$$

Where u_s is the coefficient of the amplitude of displacement for small displacement, q is the applied load, a is plate length, and D is the flexural rigidity of the plate.

The amplitude of displacement is an important factor that needs to be controlled if structural failures due to resonance are to be averted. As such design codes specific allowable limits to guide serviceability conditions. The purpose of this work is to develop a general limit state amplitude expression for the analysis of thin rectangular plates, and using the polynomial shape profiles formulate specific expressions for twelve plate types. These expressions will establish a guide on the safe limit of the amplitude of displacement based on the predicted values from the new expressions. These expressions will be very useful to plate analysts in terms of safe analysis and valuable data generation.

II. FORMULATION OF GENERAL LIMIT STATE AMPLITUDE OF DISPLACEMENT EXPRESSION

Adah (2023) gave the stress equation for yield analysis of a thin rectangular plate as

$$\sigma_x \le \frac{f_y}{F} \quad (2)$$

Where

f_y is the yield stress of steel and F is the stress factor, given as

$$F = \sqrt{\left[1 + \frac{n_2{}^2}{n_1{}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1-\upsilon-\upsilon^2+\upsilon^3)}{2Ƨ^2}\frac{n_3{}^2}{n_1{}^2}\right]} \quad (2a)$$

$$Ƨ = \frac{b}{a} \quad \text{is the aspect ratio} \quad (3)$$

υ is Poisson ratio, and n_1, n_2, and n_3 are given as

$$n_1 = \frac{\partial^2 h_x}{\partial R^2} * h_y + \frac{\upsilon}{Ƨ^2} * h_x * \frac{\partial^2 h_y}{\partial Q^2} \quad (4)$$

$$n_2 = \upsilon\left(\frac{\partial^2 h_x}{\partial R^2} * h_y\right) + \frac{1}{Ƨ^2}\left(h_x * \frac{\partial^2 h_y}{\partial Q^2}\right) \quad (5)$$

Stresses in a two-dimensional plane as given by Ibearugbulem (2017) are

$$\sigma_x = -\frac{EZ}{(1-\upsilon^2)}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right) \quad (7)$$

$$\sigma_y = -\frac{EZ}{(1-\upsilon^2)}\left(\upsilon\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \quad (8)$$

$$\tau_{xy} = -\frac{EZ(1-\upsilon)}{2(1-\upsilon^2)}\left(\frac{\partial^2 w}{\partial x \partial y}\right) \quad (9)$$

Consider a plate loaded uniaxially along the x-axis, and assuming that the effect of σ_y, and τ_{xy} are negligible, then only stress along the x-axis exists.

The displacement shape function, w, is given as

$$w = Ah \quad (10)$$

where A is the amplitude of deflection and h is the shape profile of the plate based on the plate type.
In non-dimensional parameters,

$$x = aR,\ y = bQ,\ z = St,\ 0 \le R \le 1, 0 \le Q \le 1 \quad (11)$$

Substitute Equations (10) and (11) into Equations (7) yields

$$\sigma_x = -\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right) \quad (12)$$

Where A is the amplitude of deflection.
Therefore, substituting Equation (12) into Equation (1), yields

$$-\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right) \le \frac{f_y}{F} \quad (13)$$

This can be rewritten as

$$-\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h_x}{\partial R^2} * h_y + \frac{\upsilon}{\text{Ƨ}^2} * h_x * \frac{\partial^2 h_y}{\partial Q^2}\right) \leq \frac{f_y}{F} \qquad (14)$$

Substituting for Z from Equation (11) and referring to Equation (4) makes Equation (14) becomes

$$-\frac{EAStn_1}{(1-\upsilon^2)a^2} \leq \frac{f_y}{F} \qquad (15)$$

Making A the subject of the Equation yields

$$A \leq \frac{-f_y(1-\upsilon^2)a^2}{FEStn_1} \qquad (16)$$

Equation (16) is the limiting Equation for the amplitude of deflection. Therefore, $A = A_{ls}$
And S is equal to 0.5 because it occurs at the extreme fiber of the plate.
Therefore Equation (16) becomes Equation (17)

$$A_{ls} \leq \frac{-f_y(1-\upsilon^2)}{0.5FEn_1}\frac{a^2}{t} \qquad (17a)$$

Rewritten as

$$A_{ls} \leq \mu\frac{a^2}{t} \qquad (17b)$$

Where

$$\mu = \frac{-f_y(1-\upsilon^2)}{0.5FEn_1} \qquad (18)$$

Equation (17) is the general limit state amplitude of deflection for failure analysis and is presented in Table 2.

A. FORMULATION OF SPECIFIC LIMIT STATE AMPLITUDE EQUATIONS FOR THE VARIOUS PLATE TYPE

The limit state amplitude of deflection equations for plate types is derived as follows using the shape profiles in Table 1.

Table 1: The polynomial displacement shape profiles

Plate Type	Shape Profile, h
SSSS	$(R-2R^3+R^4)(Q-2Q^3+Q^4)$
CCCC	$(R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$
CSSS	$(R-2R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
CSCS	$(R-2R^3+R^4)(Q^2-2Q^3+Q^4)$
CCSS	$(1.5R^2-2.5R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
CCCS	$(1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$
SSFS	$(R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
SCFS	$(1.5R^2-2.5R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CSFS	$(R-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
CCFS	$(1.5R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
SCFC	$(R^2-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CCFC	$(R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

Source: Ibearugbulem et al.(2014)

Using SSSS plate, type

The shape profile from Table 1 is

$$h = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) = h_x * h_y \quad (19)$$

$$\frac{\partial^2 h}{\partial R^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y = (-12R^2 + 12R^2)(Q - 2Q^3 + Q^4) \quad (20a)$$

$$\frac{\partial^2 h}{\partial Q^2} = h_x * \frac{\partial^2 h_y}{\partial Q^2} = (R - 2R^3 + R^4)(-12Q + 12Q^2) \quad (20b)$$

Substitute Equations (20) into Equations (11) yields

$$n_1 = \left[(-12R^2 + 12R^2)(Q - 2Q^3 + Q^4) + \frac{\upsilon}{\mathcal{Z}^2}(R - 2R^3 + R^4)(-12Q + 12Q^2)\right] \quad (21)$$

Substitute these values of R and Q in Equations (21) at the point of maximum deflection, R = Q = 0.5. we have

$$n_1 = \left(-0.9375 - \frac{0.9375\upsilon}{\mathcal{Z}^2}\right) \quad (22)$$

For n_2 and n_3 we have,

$$\frac{\partial^2 h}{\partial R \partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} = (-6R^2 + 4R^3)(-6Q^2 + 4Q^3) \quad (23)$$

Substitute Equations (20) into Equations (5) yields

$$n_2 = \left[\upsilon(-12R^2 + 12R^2)(Q - 2Q^3 + Q^4) + \frac{1}{\mathcal{Z}^2}(R - 2R^3 + R^4)(-12Q + 12Q^2)\right] \quad (24)$$

At the point of maximum deflection, R= Q = 0.5. Then Equation (24) becomes

$$n_2 = \left(-0.9375\upsilon - \frac{0.9375}{\mathcal{Z}^2}\right) \quad (25)$$

Also substituting Equation (23) into Equation (6) yields

$$n_3 = (1 - 6R^2 + 4R^3)(1 - 6Q^2 + 4Q^3) \quad (26)$$

At the point of maximum deflection, R= Q = 0.5. Then Equation (26) becomes

$$n_3 = 0 \quad (27)$$

Substituting Equations (22), (25) and (27) into Equation (2) yields

$$F = \left[1 + \frac{\left(-0.9375\upsilon - \frac{0.9375}{\mathcal{Z}^2}\right)^2}{\left(-0.9375 - \frac{0.9375\upsilon}{\mathcal{Z}^2}\right)^2} - 2\upsilon\frac{\left(-0.9375\upsilon - \frac{0.9375}{\mathcal{Z}^2}\right)}{\left(-0.9375 - \frac{0.9375\upsilon}{\mathcal{Z}^2}\right)}\right]^{\frac{1}{2}} \quad (28)$$

Now substituting Equations (22) and (28) into Equation (18) yields

$$u = \frac{-2f_y(1-\upsilon^2)\mathcal{Z}^2}{E(-0.9375\mathcal{Z}^2 - 0.9375\upsilon)} * \frac{1}{\sqrt{\left[1 + \frac{(-0.9375\mathcal{Z}^2\upsilon - 0.9375)^2}{(-0.9375\mathcal{Z}^2 - 0.9375\upsilon)^2} - 2\upsilon\frac{(-0.9375\mathcal{Z}^2\upsilon - 0.9375)}{-0.9375\mathcal{Z}^2 - 0.9375\upsilon}\right]}} \quad (29)$$

Equation (29) is the equation for the coefficient of limit state amplitude of deflection.

Substituting this equation into Equation (17b) yields the limit state amplitude equation for the SSSS plate as Equation (30)

$$A_{ls} \leq \frac{-2f_y(1-\upsilon^2)ƨ^2}{E(-0.9375ƨ^2 - 0.9375\upsilon)} * \frac{1}{\sqrt{\left[1 + \frac{\left(-0.9375ƨ^2\upsilon - 0.9375\right)^2}{\left(-0.9375ƨ^2 - 0.9375\upsilon\right)^2} - 2\upsilon \frac{\left(-0.9375ƨ^2\upsilon - 0.9375\right)}{-0.9375ƨ^2 - 0.9375\upsilon}\right]}} * \frac{a^2}{t} \quad (30)$$

Similarly, the other plate types based on the shape profiles in Table 1 were resolved, and the results are presented in Table 3 for the twelve plate types under consideration.

B. NUMERICAL EXAMPLE

For these analyses, the numerical parameters used are as follows. Considering a structural steel of grade A36:
f_y = 250MPa, E = 200000MPa, a = 1m, υ = 0.3, t = 0.02m.
Now substituting these parameters in Equations (29) and (30) for the SSSS plate yields the numerical results of the limit state coefficient of the amplitude of displacement presented in column 2 of Table 4, and those of limit state amplitude of displacement as presented in column 3 of Table 4.

BI. RESULTS AND DISCUSSIONS

The general limit state amplitude of displacement expression from this work is presented in Table 2.

Table 2: General Limit State Amplitude of Displacement Equation

SN	DESCRIPTION	EQUATIONS
1	Limit State Amplitude of Displacement	$A_{ls} \leq \frac{-f_y(1-\upsilon^2)}{0.5EFn_1}\frac{a^2}{t}$

This expression is applied to all edge conditions of plates. Equation (30) presented in Table 2 indicates that to avoid failure, the amplitude of displacement should be less than the value of the RHS of the equation. The equation states clearly that the amplitude of displacement varies directly proportional to the square of the length of the plate, a, and inversely proportional to the thickness of the plate, t. This means, the longer the length of the plate, the higher the amplitude of displacement. This is logical to common sense. Because if the span of any structural element either beam or plate is long the displacement or deflection is bound to be high. The common remedy to deal with longer spans is usually to increase the depth (or thickness) of the element. The implication of the equation also is that the thicker the plate the lesser the amplitude of displacement, and the thinner the the plate, the higher the amplitude of displacement. This is in line with the preceding statement, and correct for all practical situations of structural analysis and design. More so, the degree of the amplitude of displacement is a function of the properties of the material also. These properties are expressed in the equation as the Poisson ratio, υ, and the young modulus of elasticity. This is true in practice. Because the different types of materials possess different strengths due to each material's properties. For instance, steel behaves differently from wood. This shows the adequacy of this new expression. In addition, the equation also, shows that the level of amplitude of displacement of a structural material such as a plate is a function of the stress the material can withstand.

Applying this general expression to the various plate types based on the edge conditions led to the formulation of the specific amplitude of displacement equations shown in Table 3. These equations will help to analyze each plate type easily, thereby saving time and energy. This gives more information about the individual plate types. Finally, the numerical application carried out yields the results of the coefficients of the amplitude of displacement and the limit state displacement itself as presented in Table 4 for the twelve plates considered in this work. This will be of immense help to analysts and designers of plates.

With this simplification, the implication is that to determine the amplitude of displacement of mild steel, you need just to specify the length and thickness of the plate and the edge support at four edges such as CCCC or SSSS, etc. This has made things easy and this work will add much value to the knowledge and will save time and resources for experts in this field especially those in aerospace and naval architecture. And based on the implications derived from this new equation that is in line with practical structural behavior, it means the new equations are adequate for determining the limit state amplitude of displacement.

Table 3: Limit State Amplitude of Deflection Equations for Rectangular Plates of Different Pate Types

Plate Type	$A_{ls} \leq u\frac{a^2}{t}; \quad \text{Ƨ} = \frac{b}{a};$
	$u = \frac{-f_y(1-\upsilon^2)}{0.5EFn_1}$
SSSS	$\frac{-2f_y(1-\upsilon^2)\text{Ƨ}^2}{E(-0.9375\text{Ƨ}^2-0.9375\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.9375\text{Ƨ}^2\upsilon-0.9375)^2}{(-0.9375\text{Ƨ}^2-0.9375\upsilon)^2}-2\upsilon\frac{(-0.9375\text{Ƨ}^2\upsilon-0.9375)}{-0.9375\text{Ƨ}^2-0.9375\upsilon}\right]}}$
CCCC	$\frac{-2f_y(1-\upsilon^2)\text{Ƨ}^2}{E(-0.0625\text{Ƨ}^2-0.0625\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.0625\text{Ƨ}^2\upsilon-0.0625)^2}{(-0.0625\text{Ƨ}^2-0.0625\upsilon)^2}-2\upsilon\frac{(-0.0625\text{Ƨ}^2\upsilon-0.0625)}{(-0.0625\text{Ƨ}^2-0.0625\upsilon)}\right]}}$
CSSS	$\frac{-2f_y(1-\upsilon^2)\text{Ƨ}^2}{E(-0.375\text{Ƨ}^2-0.46875\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.375\text{Ƨ}^2\upsilon-0.46875)^2}{(-0.375\text{Ƨ}^2-0.46875\upsilon)^2}-2\upsilon\frac{(-0.375\text{Ƨ}^2\upsilon-0.46875)}{(-0.375\text{Ƨ}^2-0.46875\upsilon)}\right]}}$
CSCS	$\frac{-2f_y(1-\upsilon^2)\text{Ƨ}^2}{E(-0.1875\text{Ƨ}^2-0.3125\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.1875\text{Ƨ}^2\upsilon-0.3125)^2}{(-0.1875\text{Ƨ}^2-0.3125\upsilon)^2}-2\upsilon\frac{(-0.1875\text{Ƨ}^2\upsilon-0.3125)}{(-0.1875\text{Ƨ}^2-0.3125\upsilon)}\right]}}$

CCSS	$\frac{-2f_y(1-\upsilon^2)ƨ^2}{E(-0.1875ƨ^2-0.1875v)} * \frac{1}{\sqrt{\left[1+\frac{(-0.1875ƨ^2v-0.1875)^2}{(-0.1875ƨ^2-0.1875\upsilon)^2}-2\upsilon\frac{(-0.1875ƨ^2\upsilon-0.1875)}{(-0.1875ƨ^2-0.1875\upsilon)}+\frac{0.0001220703125ƨ^2(1-\upsilon-\upsilon^2+\upsilon^3)}{(-0.1875ƨ^2-0.1875\upsilon)^2}\right]}}$
CCCS	$\frac{-2f_y(1-\upsilon^2)ƨ^2}{E(-0.09375ƨ^2-0.125\upsilon)} * \frac{1}{\sqrt{\left[1+\frac{(-0.09375ƨ^2\upsilon-0.125)^2}{(-0.09375ƨ^2-0.125\upsilon)^2}-2\upsilon\frac{(-0.09375ƨ^2\upsilon-0.125)}{(-0.09375ƨ^2-0.125\upsilon)}\right]}}$
SSFS	$\frac{0.5f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$
SCFS	$\frac{f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{\left[1+\upsilon^2-2\upsilon^2+\frac{0.0008680555486(1-\upsilon-\upsilon^2+\upsilon^3)}{ƨ^2}\right]}}$
CSFS	$\frac{1.666666667f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$
CCFS	$\frac{3.333333333f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{\left[1+\upsilon^2-2\upsilon^2+\frac{0.0008680555556(1-\upsilon-\upsilon^2+\upsilon^3)}{ƨ^2}\right]}}$
SCFC	1.5 $\frac{1.5f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$
CCFC	$\frac{5f_y(1-\upsilon^2)}{E}\frac{1}{\sqrt{[1+\upsilon^2-2\upsilon^2]}}$

Table 4: Values of the Coefficients of Limit State of Amplitude of Deflection (u),

Plate Type	$A_{ls} \leq u\frac{a^2}{t}$;	
SSSS	0.00157762	0.078881
CCCC	0.02366432	1.183216
CSSS	0.00349423	0.174712
CSCS	0.00581897	0.290949
CCSS	0.00788442	0.394221
CCCS	0.01344878	0.672439
SSFS	0.00059621	0.029811
SCFS	0.00119206	0.059603
CSFS	0.00198737	0.099369
CCFS	0.00397354	0.198677
SCFC	0.00178864	0.089432
CCFC	0.00596212	0.298106

IV. CONCLUSION

The current study has derived the general limit state amplitude of displacement for analysis of thin isotropic rectangular plates. Also, the work has derived the specific amplitude of displacement equations for twelve plate types. These equations based on the discussions have been shown to conform to all practical behavior of structural materials, especially plates. The study has made it easy to predict the amplitude of displacement of plates once the length of the plate and its thickness is known with the plate type determined. The work will give analysts and designers of plates easy data to determine the limit of the safety of plate structures based on the amplitude of displacement, since the higher the amplitude of displacement, the higher the chances of failure or severity of damage. Hence, the conclusion is that these new equations are adequate for thin rectangular plate analysis for the amplitude of displacement and that the approach is simpler, understandable, and reproducible.

REFERENCES

Adah, E. I., (2016). "Development of Computer Programs for Analysis of Single Panel and Continuous Rectangular Plates". M. Eng. Thesis submitted to Postgraduate School, Federal University of Technology Owerri.

Adah, (2023). Stability and Dynamic Analysis of Thin Rectangular Plates under Large Deflection. A PhD Thesis Submitted to the Postgraduate School, Federal University of Technology Owerri

Chajes, A., (1974). "Principles of Structural Stability Theory". London: Prentice-Hall Inc (1974).

Chakraverty, S. (2009). "Vibration of Plates". (Taylor & Francis Group, London.

Enem J. I., (2018). "Geometrically Nonlinear Analysis of Isotropic Rectangular Thin Plates Using Ritz Method". Ph.D. Thesis: Department of Civil Engineering, Federal University of Technology Owerri, Nigeria.

Ibearugbulem, O.M., Ezeh, J.C., Ettu, L. O., (2014). "Energy Methods in Theory of Rectangular Plates: Use of Polynomial Shape Functions". Owerri, Liu House of Excellence Ventures, (2014).

Ibearugbulem, O. M.(2012). "Application of direct variational principles in elastic stability of rectangular flat thin plates". Ph.D thesis submitted to Postgraduate school, Federal University of Technology, Owerri.

Ibearugbulem, O. M. (2017). "Lecture Note on Large deflection analysis of continuum". Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria. Unpublished, 2017.

Iyengar, N. G., (1988). "Structural Stability of Columns and Plate". Chichester, Ellis Horwood (1988).

Levy, S., (1942). "Bending of rectangular plates with large deflections". Technical notes: National Advisory Committee for Aeronautics (NACA), N0. 846 (1942).

Oguaghamba, O. A., Ezeh, J. C., Ibearugbulem, O.M., Ettu, L. O., (2015). "Buckling and Postbuckling Loads Characteristics of All Edges Clamped Thin Rectangular Plate". The International Journal of Engineering and Science, 4(11), 55-61, (2015).

Oguaghamba, O. A., (2015). "Analysis of buckling and postbuckling loads of Isotropic thin rectangular plates". Ph.D. Thesis: Department of Civil Engineering, Federal University of Technology, Owerri Nigeria.

Onodagu, P. D.(2018) "Nonlinear Dynamic Analysis of Thin Rectangular Plates using Ritz Method" Ph.D. Thesis, Federal Univ of Tech, Owerri, Nigeria.

Onwuka, D. O., Ibearugbulem O. M. and Adah, E. I., (2016). "Stability analysis of axially compressed SSSS & CSCS plates using Matlab programming". International Journal of Science and Technoledge, 4(1), 66-71, (2016).

Szilard, R. (2004). "Theories and Application of Plate Analysis". New Jersey: John Wiley & Sons. Timoshenko, S. P., Woinowsky-Krieger, S. (1959). "Theory of plates and shells", 2^{nd} ed, McGraw-Hill, New York, (1959).

Ugural, A.C. & Fenster, S.K. (2003). "Advanced Strength & Applied Elasticity". 4th ed., New Jersey: Prentice Hall.

Ventsel, E and Krauthamer, T., (2001). "Thin Plates and Shells: Theory, Analysis and Application". Marcel Dekker, New York (2001).

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https//dosits.org>science>sound>characterise-sounds

www.sciencedirect.com>topics>engineering